

CC 3.11, Fall 2007, Professor Meagher

HW1: Read pp 40-45. Do pp 45-47, 1 -15 (odds)

1. Muchos Mangos

There are 5 layers with approximately 18 mangos each, for a total of 90

3. Alternative Lock

With five CDs, each can have its own shelf. With one more CD, some shelf must have at least two according to the Pigeonhole Principle.

5. For the Birds

The Pigeonhole Principle states that if you have more pigeons than you have pigeonholes and every pigeon must be in some pigeonhole, then there must be at least one pigeonhole with more than one pigeon.

7. Order please

States in the United States, honest congressmen, cars, telephones on the planet, people, grains of sand.

9. Twenty-nine is fine

Two possible candidates: first 29 is prime. Second, 29 happens to be the sum of three consecutive squares, $29 = 4 + 9 + 16$. (Lest the number 27 feel left out, it should be noted that $27 = 3 \times 3 \times 3$ is a perfect cube.)

11. Many fold

To get started, let's estimate the width of an ordinary piece of paper by noting that packages of 200 sheets of paper are more than half an inch thick. Now, after one folding, the paper is twice the original thickness. After two foldings, the paper is $4 = 2^2$ times as thick. After 50 foldings the paper will be 2^{50} times as thick. The resulting paper is then $2^{50}/400$ inches thick. (That's more than 2.8×10^{13} in. and more than 40 million miles!)

13. For the Birds

There must be some hole containing more than one pigeon. In the hairy-bodies question, the six billion people in the world play the role of the pigeons, and the 400 million hairs play the role of the holes. Just as there are at least two pigeons sleeping in the same hole, there are necessarily two people with the same total number of body hairs.

15. The Last One

19, 58, 29, 88, 44, 22, 11, 34, 17, 52, 26, 13, 40, 20, 10, 5, 16, 8, 4, 2, 1

The sequences for 11 and 22 are within the sequence above.

30, 15, 46, 23, 70, 35, 106, 53, 160, 80, 40, 20, 10, 5, 16, 8, 4, 2, 1